1 Introduction

The centralized and decentralized control of large-scale networks has received considerable attention. Although the decentralized schemes have been widely employed due to their computationally efficient algorithms, but some crucial trade-offs must be carefully addressed. The critical robustness and practical feasibility aspects of the decentralized controllers, in the presence of interconnected harmful responses, need to be thoroughly investigated. Note that the decentralized control of a multiphysics electromagnetomechanical-fluid network would be challenging to deal with. Many uncertainties involved with various coupled multidisciplinary components potentially result in cumbersome computational burden in addition to the lack of robustness. Therefore, a robust, computationally efficient, and practically feasible decentralized controller is needed to be examined and then implemented in such a large-scale multi-agent network.

The so-called “smart valves” network is widely utilized in many critical infrastructures including, but not limited to, municipal piping systems, oil and gas fields, petrochemical plants, and defense industries. Such a multiphysics flow distribution network deals with various aspects of fluid mechanics, electromagnetics, and electromechanical components. The economic and even social impact of failure of such an essential network, for each application addressed previously, would be expected to be dramatic, and therefore, a robust and practically feasible control scheme is required to mitigate the effects of the harmful dynamic responses; in the presence of enormous uncertainties involved with such a large-scale network. In particular, the system we study here is a network of two dynamically interconnected bidirectional solenoid actuated butterfly valves operating in series, as shown in Fig. 1.

Note that we have previously carried out [1–16] broad analytical and experimental studies from nonlinear interconnected modeling to centralized and direct decentralized adaptive control of both an isolated actuator-valve agent and a network of two solenoid actuated butterfly valves dynamically coupled in series.

It is somewhat difficult to find specific research work related to capturing and then controlling the chaotic and hyperchaotic dynamics of smart flow distribution network using the decentralized neuro-adaptive scheme. However, some efforts can be found in Refs. [17–30] addressing the control of broad electromechanical systems. Hsiao et al. [31] studied stabilization problem for a neural-network (NN) linearly interconnected system consisting a number of NN models. They established a linear difference inclusion state-space representation for the dynamics of each NN model. Subsequently, according to the decentralized control scheme, a set of Takagi–Sugeno (T–S) fuzzy controllers was synthesized to stabilize the NN linearly interconnected system. Karpenko et al. [32] employed reinforcement learning to coordinate the motions of a pair of hydraulic actuators. Yang and Yue [33] developed an adaptive observer to reconstruct unavailable state information taking advantage of the universal approximation property of NNs. They then recursively designed an observer-based decentralized adaptive fault-tolerant control strategy by combining backstepping methods with NNs, fault-tolerant control theory, and the dynamic surface control technique. Eltantawi [34] created decentralized neuro-fuzzy controller to improve the ride comfort and increase the stability for half car suspension system using the magneto-rheological damper as a semi-active device. Shi and Singh [35] studied robust decentralized adaptive controller design for interconnected systems. They considered a general representation of interconnections when the strength of the interconnections is bounded by a $p$th-order polynomial in states. Duan and Min [36] solved the decentralized state-feedback control problem for a class of large-scale stochastic high-order nonlinear systems. By generalizing NN approximation approach to this kind of systems, they completely removed the growth conditions on system nonlinearities and the power order restriction. Tong et al. [37] proposed an adaptive fuzzy decentralized backstepping output feedback control to be utilized for a class of uncertain large-scale stochastic nonlinear systems without the measurements of the states. The fuzzy logic systems are used in approximating the unknown nonlinear functions, and a fuzzy state
observer is designed for estimating the unmeasured states. They revealed that the proposed control approach can guarantee that all the signals of the resulting closed-loop system are semi-globally uniformly ultimately bounded in probability, and the observer errors and the output of the system converge to a small neighborhood of the origin by choosing appropriate design parameters. Zhang et al. [38] investigated decentralized adaptive control for a class of discrete-time nonlinear hidden leader–follower multi-agent systems. They provided rigorous mathematical proofs to reveal that the hidden leader agent tracks the desired reference signal, all the follower agents follow the hidden leader agent, and the closed-loop system eventually achieves strong synchronization in the presence of strong couplings. Skworew et al. [39] considered development of a methodology for an online energy and leakage management in water distribution systems, formulated within a model predictive control (MPC) framework. The approach involved calculation of control actions, i.e., time schedules for pumps, valves, and sources, to minimize the costs associated with energy used for water pumping and treatment and water losses due to leakage, while satisfying all operational constraints.

Negenborn et al. [40] considered the control of large-scale transportation networks, like road traffic networks, power distribution networks, water distribution networks, etc. Control of these networks is often not possible from a single point by a single intelligent control agent; instead, control has to be performed using multiple intelligent agents. They considered multi-agent control schemes in which each agent employs a model-based predictive control approach. Coordination between the agents was used to improve decision making. Bottura and Caceres [41] analyzed river water quality systems with a serial interconnection structure with the objective of multivariable control. For the resulting block triangular system, a control design methodology was proposed in such a way to permit that from multivariable input–output data for such a type of system, both the identification by a subspace methodology previously developed be made as the corresponding decentralized control. Joseph-Duran et al. [42] presented an output-feedback control strategy for pollution mitigation in combined sewer networks. Their strategy provided means to apply model-based predictive control to large-scale sewer networks, in spite of the lack of measurements at most of the network sewers. To cope with uncertainty in system disturbances due to the stochastic water demand/consumption and optimize operational costs, Grosso et al. [43] proposed three stochastic model predictive control approaches, namely, chance-constrained MPC, tree-based MPC, and multiple-scenario MPC. Fambrini and Ocampo-Martinez [44] designed and tested MPC strategies for the global centralized and decentralized control of drinking water networks (DWN). Tests have been performed in order to highlight the advantages of having a partition of a complex network in several subsystems. Barcelli et al. [45] proposed an automatic model decomposition approach for decentralized model predictive control of DWNs. For a given DWN, the proposed algorithm partitioned the network in a set of subnetworks by taking advantage of the topology of the network, of the information about the use of actuators, and of system management heuristics.

We briefly represent the nonlinear interconnected modeling of two agents (for completeness) in addition to the initial conditions and crucial parameters resulted in the harmful dynamic responses. Then, the decentralized neuro-adaptive control scheme is formulated to be used in suppressing the interconnected chaotic and hyperchaotic dynamics of two agents. The results are thoroughly compared with those of the direct decentralized adaptive one to reveal crucial trade-offs and then yield a robust, computationally efficient, and practically feasible control scheme.

2 Mathematical Modeling

We have previously formulated [1–5] the interconnected analytical model of two agents operating in series and briefly represent here for completeness. The small-scale network being studied here is a set of two symmetric butterfly valves driven by bidirectional solenoid actuators (Fig. 1) through rack and pinion arrangements. Note that utilizing the rack and pinion mechanism provides the kinematic constraint helping us to formulate the coupled multiphysics model of two agents.

Developing such an interconnected multiphysics model undoubtedly needs some simplifying assumptions to be applied. Among those assumptions reported earlier [1–5], the most important one is to assume the dominant laminar flow for both the coupled valves in order to avoid the numerical difficulties involved with a turbulent regime. However, a critical issue needs to be addressed with respect to the validity of such an assumption. The values of pipe diameters and flow mean velocity given in Table 1 expectedly indicate the existence of the turbulent regime invalidating the assumption we have made. From another aspect, the analytical formulas of the flow loads, including the hydrodynamic and bearing torques, were formulated based on the assumption of laminar flow [46]. To address the issues discussed previously, we have carried out experimental work [47], shown in Fig. 2, to measure the sum of the hydrodynamic and bearing torques as the most affecting loads on the valves and subsequently, the dynamics of the actuators. The experiment yielded the total torque for the inlet velocity of $v \approx 2.7$ (m/s) and valve diameter of $D_v = 2$ in reasonably validating the laminar flow assumption.
We proposed an effective method of “coupled resistances” [1–5] to model the interconnected agents by utilizing the mass continuity and postdiffusion electromagnetic principles. Note that the inlet and outlet pressures were supposed to be known. As can be observed in Fig. 1(b), the valves are modeled as changing resistors

\[ R_{ai}(x_i) = \frac{e_i}{p_i x_i^3 + q_i x_i^2 + a_i x_i + \gamma_i}, \quad (i = 1, 2) \]  

where, \( R_{a1} \) and \( R_{a2} \) indicate the resistances of the upstream and downstream valves, respectively, and \( e_1 = 7.2 \times 10^5, \; p_1 = 461.9, \; q_1 = -405.4, \; a_1 = -1831, \; \gamma_1 = 2207, \; c_2 = 4.51 \times 10^3, \; p_2 = 161.84, \; q_2 = -110.53, \; a_2 = -695.1, \; \) and \( \gamma_2 = 807.57 \) for two different valves’ diameters. Also, the flow between the valves in addition to the sudden contraction is modeled as constant resistors based on the Hagen–Poiseuille and Borda–Carnot formulas [48,49]

\[ R_{L1} = \frac{128 \mu L_1}{\pi D_{in}^2}, \quad (i = 1, 2) \]  

where \( K_{con} = 0.5(1 - \beta^2) \sqrt{\sin(\beta/2)}, \; \beta \) indicates the ratio of minor and major diameters \((D_{o2}/D_{i2})\), \( \theta \) is the angle of approach (the pipe contraction angle), \( \mu \) stands for the fluid dynamic viscosity, \( D_{i1} \) and \( D_{i2} \) are the upstream and downstream valves’ diameters, respectively, \( L_1 \) and \( L_2 \) indicate the pipe lengths before and after contraction, and \( R_{L1} \) and \( R_{L2} \) are the constant resistances. Therefore, two valves operating in series can be modeled as a set of five resistors, leading us to derive mathematical expressions of the pressures after and before the upstream and downstream valves, respectively, as follows [1–5]:

\[ p_1 = \frac{R_{a2} P_{in} + R_{a1} P_{out} + R_{con}(R_{L1} + R_{L2} + R_{con} q_i) q_i}{(R_{L1} + R_{a2})} \]  

\[ p_2 = \frac{R_{a2} P_{in} + R_{a1} P_{out} - R_{L2} (R_{L1} + R_{L2} + R_{con} q_i) q_i}{(R_{L1} + R_{a2})} \]  

where \( q_i \) is the volumetric flow rate. These interconnected pressures were used in developing both the coupled hydrodynamic and bearing torques [1–5]

\[ T_{h1} = \left( a_1 x_1^3 e_i x_1^2 + c_1 e_i x_1 \right) \left( P_{in} - P_{out} \right) \]  

\[ T_{h2} = \left( a_1 x_2^3 e_i x_2^2 + c_1 e_i x_2 \right) \left( P_{in} - P_{out} \right) \]  

\[ T_{h1} = C_1 \Delta P_{1}(R_{a1}, R_{a2}, R_{L1}, R_{L2}, R_{con}) \]  

\[ T_{h2} = C_2 \Delta P_{2}(R_{a1}, R_{a2}, R_{L1}, R_{L2}, R_{con}) \]  

where \( a_1 = 0.4249, \; a_1' = 0.1022, \; b_1 = -18.52, \; b_1' = -17.0795, \; c_1 = -7.823 \times 10^{-4}, \; c_1' = -2 \times 10^{-4}, \; d_1 = -1.084, \; d_1' = -1.0973, \; C_1 = C_2 = 0.5A_{li}D_{in}, \Delta P_{1} = P_{in} - P_{out}, \Delta P_{2} = P_{2} - P_{out}, \) and \( P_{in} \) and \( P_{out} \) are the inlet and outlet pressures given, respectively. Note that \( D_2 \) is the diameter of the valve and \( \mu \) stands for the friction coefficient of the bearing area. We have previously established that the hydrodynamic torque acts as a helping load pushing the valve to be closed and is typically effective for when the valve angle is lower than 60 deg [1–16]; the effective range was experimentally examined [47] confirming the helping behavior of the hydrodynamic torque by presenting positive values. The bearing torque, due to its friction-based nature, always acts as a resisting load. Using these tools, we could formulate the sixth-order inter-connected model of two agents as follows:

\[ \dot{z}_1 = z_2 \]  

[Fig. 2] (a) The experimental work setup and (b) the experimentally measured total flow loads.

**Table 1** The system parameters

| \( \rho \) | 1000 (kg/m³) | \( \nu \) | 3 (m/s) |
| \( L_{i1} \) | 0.104 \times 10^{-6} (kg/m²) | \( N_i \) | 3300 |
| \( N_i \) | 3300 | \( C_{Li1,2} \) | 1.56 \times 10^6 (H⁻¹) |
| \( D_{i1} \) | 0.2032 (m) | \( D_{i2} \) | 0.127 (m) |
| \( D_{i1,2} \) | 0.01 (m) | \( P_{out} \) | 2 (kPa) |
| \( k_{i1,2} \) | 60 (N·m⁻²) | \( C_{Li1,2} \) | 6.32 \times 10^6 (H⁻¹) |
| \( L_{1} \) | 2 (m) | \( L_{2} \) | 1 (m) |
| \( r_{i1,2} \) | 0.05 (m) | \( \theta \) | 90 deg |
| \( P_m \) | 256 (kPa) | \( \rho_{i1,2} \) | 0.1 (m) |
| \( \mu_f \) | 0.018 (kg·m⁻¹·s⁻¹) | \( \beta_i \) | 1 \times 10⁻⁷ |

\[ \dot{z}_1 = z_2 \]
\[
\dot{z}_2 = \frac{1}{J_1} \left[ \frac{r_1 C_{21} N_1^2 z_1^2}{2(C_{11} + C_{21}(g_{m1} - r_1 z_1))^2} - \frac{(P_{in} - P_{out} - (R_{L1} + R_{L2} + R_{out} q_i)) q_i}{e_i} \right] b_{21} z_2 - k_1 z_1 + \frac{(p_1 z_1^2 + q_1 z_1 + a_1 z_1 + \gamma_1)^2}{e_i} \sum_{i=1}^{4} (p_1 z_1^2 + q_1 z_1 + a_1 z_1 + \gamma_1)^2 \right] \times \left[ a_1 z_1 e^{\phi_1} + c_1 e^{\phi_1} - c_1 e^{\phi_1} \right] - C_1 \times \tanh(K z_1)
\]

(11)

\[
\dot{z}_3 = \frac{1}{J_2} \left[ \frac{r_2 C_{22} N_2^2 z_4^2}{2(C_{12} + C_{22}(g_{m2} - r_2 z_4))^2} - \frac{(P_{in} - P_{out} - (R_{L1} + R_{L2} + R_{out} q_i)) q_i}{e_i} \right] b_{22} z_5 - k_2 z_4 + \frac{(p_2 z_4^2 + q_2 z_4 + a_2 z_4 + \gamma_2)^2}{e_i} \sum_{i=1}^{4} (p_2 z_4^2 + q_2 z_4 + a_2 z_4 + \gamma_2)^2 \right] \times \left[ a_2 z_4 e^{\phi_2} + c_2 e^{\phi_2} - c_2 e^{\phi_2} \right] - C_2 \times \tanh(K z_4)
\]

(12)

\[
\dot{z}_4 = z_3
\]

(13)

\[
\dot{z}_5 = \frac{1}{J_2} \left[ \frac{r_2 C_{22} N_2^2 z_6^2}{2(C_{12} + C_{22}(g_{m2} - r_2 z_6))^2} - \frac{(P_{in} - P_{out} - (R_{L1} + R_{L2} + R_{out} q_i)) q_i}{e_i} \right] b_{22} z_5 - k_2 z_4 + \frac{(p_2 z_6^2 + q_2 z_6 + a_2 z_6 + \gamma_2)^2}{e_i} \sum_{i=1}^{4} (p_2 z_6^2 + q_2 z_6 + a_2 z_6 + \gamma_2)^2 \right] \times \left[ a_2 z_6 e^{\phi_2} + c_2 e^{\phi_2} - c_2 e^{\phi_2} \right] - C_2 \times \tanh(K z_6)
\]

(14)

\[
\dot{z}_6 = \frac{1}{J_2} \left[ \frac{r_2 C_{22} N_2^2 z_6^2}{2(C_{12} + C_{22}(g_{m2} - r_2 z_6))^2} - \frac{(P_{in} - P_{out} - (R_{L1} + R_{L2} + R_{out} q_i)) q_i}{e_i} \right] b_{22} z_5 - k_2 z_4 + \frac{(p_2 z_6^2 + q_2 z_6 + a_2 z_6 + \gamma_2)^2}{e_i} \sum_{i=1}^{4} (p_2 z_6^2 + q_2 z_6 + a_2 z_6 + \gamma_2)^2 \right] \times \left[ a_2 z_6 e^{\phi_2} + c_2 e^{\phi_2} - c_2 e^{\phi_2} \right] - C_2 \times \tanh(K z_6)
\]

(15)

where, \(b_{ij}\) indicates the equivalent torsional damping, \(k_i\) is the equivalent torsional stiffness, \(V_i\) stands for the supply voltage, \(r_i\) indicates the radius of the pinion, \(C_{ij}\) and \(C_{2i}\) are the reluctances of the magnetic path without air gap and that of the air gap, respectively, \(N_i\) stands for the number of coils, \(g_{m1}\) is the nominal air gap, \(J_i\) indicates the polar moment of inertia of the valve’s disk, and \(R_i\) is the electrical resistance of coil. \(z_1 = x_1, z_2 = x_2, z_3 = x_3\), and \(z_4 = x_1, z_5 = x_2, z_6 = x_3\) indicate the upstream valve’s rotation angle, angular velocity, and actuator current, respectively. \(z_4 = x_1, z_5 = x_2, z_6 = x_3\) stand for the downstream valve’s rotation angle, angular velocity, and actuator current, respectively. The network parameters are listed in Table 1. Note that it is important to thoroughly study the existence and uniqueness of solution of the model formulated. Equations (10)–(15) can be lumped as

\[
\dot{Z} = F(Z)
\]

(16)

where \(Z = [z_1, z_2, z_3, z_4, z_5, z_6]^T\).

**Theorem 1.** Assume \(F(Z)\) is a continuous function [50] in a region given as

\[
R = \{Z \in \mathbb{R}^6 : \|Z - Z_0\| \leq a\}, \quad a > 0
\]

(17)

Since \(F\) is continuous in a closed and bounded domain, it is necessarily bounded in \(R\):

\[
\exists K > 0 \quad \text{such that} \quad \|F(Z)\| \leq K \quad \forall Z \in R
\]

(18)

Based on the theorem, it is straightforward to conclude that \(\dot{Z} = F(Z)\) has at least one solution.

**Theorem 2.** Assume \(F\) and \(\partial F / \partial Z\) are continuous functions in \(R\) [50] defined through the existence theorem. Therefore, both the \(F\) and \(\partial F / \partial Z\) are bounded in \(R\) whereas \(F\) is Lipschitz:

\[
\exists K, L > 0 \quad \text{such that} \quad \|F(Z)\| \leq K \quad \text{and} \quad \|F(Z) - F(Y)\| \leq L\|Z - Y\| \quad \forall Z, Y \in R
\]

(19)

This indicates that \(\dot{Z} = F(Z)\) has at least one solution.

Comparing the conclusion of at most one solution with the existence theorem leads to a unique solution of \(\dot{Z} = F(Z)\). It is obvious that all \(F_\phi^{(i)}\)’ and \(\partial F_\phi^{(i)} / \partial \phi_\psi\) (\(\phi, \psi = 1, \ldots, 6\)) are continuous on a certain operational domain (open and connected set, \(D \subset \mathbb{R}^6\)) of the system; there is no singular point in the operational domain. Consequently, \(F_\phi^{(i)}\)’s and \(\partial F_\phi^{(i)} / \partial \phi_\psi\) are bounded in \(D\) resulting in a Lipschitz \(\dot{F}\) at every point \(z \in D\).

By exposing the coupled agents to the critical parameters of \(b_{ij} = \mu_i = 1 \times 10^{-7}\) along with a set of certain initial conditions, we could capture [2], for the first time, the interconnected chaotic and hyperchaotic dynamics shown in Figs. 3(6) and 3(3), respectively. The initial conditions leading to the coupled chaos and hyperchaos are initial1 = [20(deg) 0 0 20(deg) 0 0] and initial2 = [20(deg) 0 0 20(deg) 0 0], respectively. We utilized the powerful tools of Lyapunov exponents and Poincaré map to distinguish between the responses. The Lyapunov exponent is a powerful indicator to reveal the divergence rate of two nearby trajectories (valves-actuators)

\[
L_j = \lim_{t \to \infty} \lim_{\|\Delta Z_0\| \to 0} \frac{1}{t} \ln \frac{\|\Delta Z_0(\{0, t\})\|}{\|\Delta Z_0\|}
\]

(20)
The algorithm utilized in determining Lyapunov exponents can be found in Ref. [51]. One and two positive Lyapunov exponents shown in Figs. 4(a) and 4(b) indicate the chaotic and hyperchaotic dynamics of the two agents. The irregular Poincaré maps (Figs. 4(c)–4(f)) confirm the existence of the harmful responses.

Extinguishing such harmful dynamic responses, in particular with larger domains of attractions, would undoubtedly require a robust, computationally efficient, and more importantly, practically feasible control scheme. We have previously examined a centralized adaptive method [1] by yielding practically feasible control inputs although with a considerable computation time. The issue of computational burden, particularly for the large-scale networks, led us to examine the direct decentralized adaptive method [11]. The decentralized scheme’s computation time was one-sixtieth of the central one although yielded practically infeasible control inputs. Therefore, we here examine a decentralized combinatorial method to possibly fix the shortcomings of the direct decentralized adaptive one.

3 Decentralized Neuro-Adaptive Scheme

The decentralized neuro-adaptive scheme reported in Refs. [52–55] is utilized for tracking energy-efficient trajectories [3,4] defined based on the crucial initial conditions

\[ x_{i1} = \frac{\pi}{3} \tanh(10^{-4} \beta) + \frac{\pi}{9}, \text{ Initial}_1 \]  
\[ x_{i2} = \frac{\pi}{3} \tanh(10^{-4} \beta) + \frac{\pi}{90}, \text{ Initial}_2 \]

Note that the so-called “S-Shaped” trajectories are highly energy-efficient [4] and yield smooth dynamic responses avoiding the repeatedly observed dangerous phenomenon of “Water Hammering.” We rewrite the interconnected dynamic Eqs. (11) and (14) as

\[ J_i \ddot{x}_i + b_i \dot{x}_i + k_i x_i = \frac{r_i C_2 N_i^2 u_i}{2(C_{1i} + C_{2i}(g_{ui} - r_i z_i)^2)} + \frac{A_i R_{ui}}{2 \sum_{i=1}^{N} R_{ui}} \times \left[ T_{hi} - T_{hi}\tan(K z_i) \right], \quad (i = 1, 2) \]

where \( A_1 = (P_{in} - P_{out} - (R_{L1} + R_{L2} + R_{cons}) q_i), T_{hi} = a_1 z_i e^{x_i^2}, T_{hi} = a_1 z_i e^{x_i^2}, \) and \( T_{hi} = 0.5 A_i b_i D_{ui}. \)

Generally, a large-scale network containing \( N \) interconnected agents, here \( N = 2, \) can be expressed as follows:

\[ \dot{x}_{i1} = x_{i2} \]  
\[ \dot{x}_{i, u_i} = f_i(x_i, u_i) + \Delta(x_1, x_2, \ldots, x_N) \]

where \( x_i = [x_{i1}, x_{i2}, \ldots, x_{i8}]^T \) is the state vector of \( i \)th agent and \( x = [x_1^T, x_2^T, \ldots, x_N^T]^T \) indicates the full state of the whole network. \( u_i \) and \( y_i \) stand for the decentralized input and output of the \( i \)th agent, respectively; \( f_i(x_i, u_i) \) is a smooth function and \( |\Delta_i x_1, x_2, \ldots, x_N| \leq a_i (a_i > 0) \) stands for the effects of the other interconnected sets.

By defining the tracking error \( e_i = y_i - x_i \) as the error vector of the \( i \)th agent is written as \( e_i = [e_i, e_i, e_i(0)]^T \) with the time derivative of \( e_i = [e_i, e_i, e_i(0)]^T \) \((i = 1, 2)\). The error dynamics can be therefore written as

\[ e_i^{(n)} = y_i^{(n)} - x_i^{(n)} \]
\[ = \dot{x}_{i, u_i} - x_i^{(n)} \]
\[ = f_i(x_i, u_i) + \Delta(x_1, x_2, \ldots, x_N) - x_i^{(n)} \]

Using the mean value theorem [56], we have

\[ f_i(x_i, u_i) = f_i(x_i, u_i') + (u_i - u_i) f_{u_i} \]

where \( f_{u_i} = \frac{\partial f_i(x_i, u_i)}{\partial u_i} |_{u_i = u_i'}. \) Substituting Eq. (30) into Eq. (29) yields

\[ e_i^{(n)} = f_i(x_i, u_i') + (u_i - u_i) f_{u_i} + \Delta_i - x_i^{(n)} \]

Assuming \( v_i = f_i(x_i, u_i') \) which is a pseudocontrol signal, and rewriting Eq. (31) give

\[ e_i^{(n)} = v_i + (u_i - u_i') f_{u_i} + \Delta_i - x_i^{(n)} \]

Hence, the pseudocontrol \( v_i \) [52] is derived as

\[ v_i = -(a_{i,n}-1)e_i^{(n-1)} + \cdots + a_{i,1}e_i + a_{i,0}e_i + r_i^{(n)} \]

The coefficients of error terms are selected such that \( L_i(s) = s^{(n)} + a_{i,n-1}s^{(n-1)} + \cdots + a_{i,0} \) becomes Hurwitz. Substituting Eq. (33) into Eq. (32) yields

\[ e_i^{(n)} = -(a_{i,n}-1)e_i^{(n-1)} + \cdots + a_{i,1}e_i + a_{i,0}e_i + (u_i - u_i') f_{u_i} + \Delta_i \]
Therefore, the error dynamics can be expressed as follows:

\[ \dot{e}_i = A_i e_i + b_i ((u_i - u_i') f_{u_i} + \Delta_i), \quad (i = 1, 2) \]  

(35)

The matrix \( A_i \) is Hurwitz and \( b_i = [0, 1]^T \) for the two interconnected agents subject to the chaotic and hyperchaotic dynamics. The Hurwitz \( A_i \) leads to a unique positive definite \( P_i \) to be calculated through the Lyapunov equation

\[ A_i^T P_i + P_i A_i = -Q_i \]

(36)

where \( Q_i \) is a positive definite matrix.

The following decentralized neuro-adaptive control inputs developed in Ref. [52] are utilized in extinguishing the harmful responses:

\[
\begin{align*}
    u_{\text{dist}} & = \hat{B}_i^T \psi_b(z_i) - (e_i^T P_i b_i) \hat{\psi}_i (e_i^T P_i b_i) - \hat{\psi}_i \text{sgn}(e_i^T P_i b_i) \\
    & - (e_i^T P_i b_i) \hat{\Omega}_i + u_{i,b}(z_i) - \frac{N(e_i^T P_i b_i)}{2(f_i^2)^T} , \quad (i = 1, 2)
\end{align*}
\]

(37)

where \( \hat{B}_i^T \psi_b(z_i) \) indicates the radial basis neural network employed to approximate the ideal controller for the network.
\[ z_t = [x_t^T, v_t]^T, \psi_{bh}(z_t) \in \mathbb{R}^{K_b} \] stands for the NN basis vector, \( K_b \) is the NN’s number of nodes, the second term \((e_i^T P_i b_i^T) \psi_{bh}(z_t)\) is utilized in compensating for the interconnections’ non-linearities, the \( u_{i,k}(z_t) \) stands for a prior continuous controller designed in advance via heuristics or past experiences with the application of conventional control methods, and \( \zeta_{bh}, \text{sgn}(e_i^T P_i b_i^T)\), \((e_i^T P_i b_i^T)\Omega_{bh}\), and \((N(e_i^T P_i b_i^T)/2(f_i^2)^2)\) are used in dealing with uncertainties in the neural network approximation error and the network interconnections.

Also, the following neuro-based adaptation laws are employed:

\[ \dot{b}_i = -\Gamma_{bh} e_i^T P_i b_i \psi_{bh}(z_t) \] \( \dot{c}_i = \Gamma_{ci} (e_i^T P_i b_i) z_i \] \[ \dot{\xi}_i = \gamma_i |e_i^T P_i b_i| \] \[ \dot{\Omega}_{bh} = \gamma_{bh} (e_i^T P_i b_i)^2 \]

where \( \Gamma_{bh}, \Gamma_{ci}, \gamma_i, \) and \( \Omega_{bh} \) are constant adaptation gains. Note that the decentralized neuro-adaptive control and adaptation laws developed in Ref. [57] guarantee asymptotic convergence of the tracking errors to zero and also boundedness of the closed-loop network using Barbalat and the following lemmas and assumption.

**ASSUMPTION.** For each agent, the followings are valid for positive constants \( f_i^4 \) and \( H_i \):

\[ 0 < f_i^4 \leq \frac{\partial f_i(z_i, u_i)}{\partial u_i} \] \[ \frac{d}{dt} \left[ \frac{\partial f_i(z_i, u_i)}{\partial u_i} \right] \leq H_i \leq \frac{\min(Q_i)}{\max(P_i)} f_i^4 \]

where \( Q_i \) and \( P_i \) are the positive-definite matrices.

**LEMMA.** For the ith agent described in Eqs. (24)–(26) satisfying the above-mentioned assumption and also other conditions given in Ref. [58], the following inequality is valid:

\[ e_i^T Q_i e_i + e_i^T P_i f_i u_i \geq 0 \]

Therefore, by selecting the Lyapunov function as

\[ V = \sum_{i=1}^{N} e_i^T P_i e_i + \frac{1}{2} \left[ \tilde{B}_i^T \Gamma_{bh}^{-1} \tilde{B}_i + \tilde{C}_i^T \Gamma_{ci} \tilde{C}_i + \frac{\tilde{\Omega}_{bh}^2}{\gamma_{bh}^2} + \frac{\tilde{\Omega}_{ci}^2}{\gamma_{ci}^2} \right] \]

One can obtain \( \dot{V} \) as follows:

\[ \dot{V} \leq -\sum_{i=1}^{N} e_i^T Q_i e_i + e_i^T P_i f_i u_i \]

The \( \dot{V} \) indicates asymptotic converge of the tracking errors to zero, based on the lemma mentioned, in addition to the boundedness of the closed-loop network. Note that we have previously utilized the direct decentralized adaptive scheme [11] using the following control and adaptation laws:

\[ u_{i,k} = \frac{\nu_i}{\eta_i} \]

**4 Results**

Shown in Figs. 5 and 6 are the estimation processes for both the decentralized neuro- and direct-adaptive schemes suppressing the coupled chaotic dynamics. Comparing the results of both the schemes reveals the lack of robustness against uncertainties although the neuro-adaptive method seems to have slightly better performance than the direct-based one. In particular, the estimation process for the \( \eta_i \)'s of the direct-based method (Fig. 6(c)), by revealing oscillatory-like profiles and also used in Eq. (47), would potentially result in practically infeasible control inputs.

It is of great interest to observe that both the methods suppressing the coupled hyperchaotic dynamics timely converge. Figure 7 indicates that the neuro-adaptive scheme suppressing the coupled hyperchaos is considerably more robust than the chaotic case (Fig. 5). This looks interesting as the hyperchaotic network is subject to higher amplitude stochastic oscillations in comparison with the chaotic one (Fig. 3), and therefore, we expected to observe the better robustness for the neuro-adaptive scheme.
extinguishing the coupled chaotic dynamics. We also revealed [11] better robustness for the direct decentralized method suppressing the coupled hyperchaotic dynamics in comparison with the interconnected chaos. Therefore, selecting a superior performance for the robustness against uncertainties, as the first trade-off addressed earlier, between the decentralized neuro- and direct-adaptive methods looks challenging. However, it is obvious that the robustness of the centralized adaptive scheme, which we reported in Refs. [1] and [11], is significantly better than both the decentralized methods. Note that both the decentralized approaches, based on the “sufficient richness” condition [59,60], would not exactly estimate the unknown parameters such that the schemes expectedly yield values to allow the desired task to be carried out.

Shown in Fig. 8 are the control inputs and driving magnetic torques of both the neuro-adaptive and direct-based methods for the network subject to the coupled chaotic dynamics. By comparing Figs. 8(a) and 8(b), one can easily conclude that the neuro-adaptive control inputs are practically feasible in comparison with the chattering control inputs of the direct-based scheme. The same profiles of the control inputs are expected to be observed for the driving magnetic torques shown in Figs. 8(c) and 8(d). It is fairly
straightforward to conclude that implementing the control inputs and subsequently driving magnetic torques of the direct-based method in extinguishing the coupled chaotic dynamics would potentially result in failures of the coupled actuation units and gradually the whole network. Therefore, the neuro-adaptive scheme looks as a feasible tool for stabilizing the coupled chaotic network although another trade-off, the computational burden, needs to be carefully addressed.

Figure 9 presents the valves’ rotation angles and tracking errors of both the schemes used in suppressing the coupled chaos. Figures 9(a) and 9(b) reveal that the upstream and downstream valves utilizing both the methods tend to the desirable trajectories (Eq. (21)). The tracking errors of the neuro-adaptive and direct-based methods presented in Figs. 9(c) and 9(d), respectively, are negligible although the direct-based scheme shows a better performance by yielding considerably smaller tracking errors in comparison with those of the neuro-adaptive one. Note that the noisy profiles of the direct-based tracking errors look logical with respect to the chattering control inputs. Considering the practically feasible control inputs of the neuro-adaptive method and its slightly better performance, with respect to the tracking errors of the direct-based one, we may select the neuro-adaptive scheme as an efficient and powerful controller to be used in suppressing the coupled chaotic dynamics. However, this selection would be questionable with respect to both the methods’ computational times. The computation time of the direct-based scheme is one-fiftieth of the neuro-adaptive one: \( t_{\text{direct}} = 0.867 \text{s} \) and \( t_{\text{neuro}} = 40 \text{s} \). Therefore, using the neuro-adaptive method would potentially cause crucial issues by adding more agents into the network. Consequently, an important trade-off needs to be addressed between the practical feasibility of control inputs and computational burden for extinguishing the interconnected chaotic dynamics.

Shown in Fig. 10 are the control inputs and driving magnetic torques to be used in suppressing the coupled hyperchaotic dynamics of two agents. As expected, higher control inputs and subsequently driving magnetic torques, for both the schemes, need to be applied in order to extinguish the coupled hyperchaos in comparison with the interconnected chaotic dynamics (Fig. 8). This looks logical with respect to the larger domains of attractions of the hyperchaotic responses than those of the chaotic ones. It is of a great interest to observe that the control inputs of both the schemes (Figs. 10(a) and 10(b)), in the presence of the hyperchaotic responses having the higher amplitude stochastic oscillations, are almost practically feasible. However, the neuro-based method yields slightly smoother control inputs, except within small transient domains limited up to 3 s, than those of the direct-based one. The same profiles are logically expected to be observed for the driving magnetic torques shown in Figs. 10(c) and 10(d). Therefore, the practical feasibility of control inputs would not be a critical issue for selecting between the neuro-adaptive and direct-based methods. Hence, the other trade-offs need to be carefully considered.

Figure 11 presents the valves’ rotation angles and tracking errors of both the neuro-adaptive and direct-based schemes utilized in suppressing the coupled hyperchaos. Despite the agents’ motions subject to the coupled chaotic dynamics (Figs. 9(a) and
we are able to distinguish considerable differences for the valves’ rotation angles shown in Figs. 11(a) and 11(b). It is fairly straightforward to conclude that the neuro-adaptive scheme outperforms the direct-based one for perfect tracking of the desirable trajectories (Eq. (22)). Although both the valves, using the direct-based scheme, finally reach to the angles targeted ($\theta_{\text{ref}} = 62$ deg), the direct-based method is inefficient for wide ranges of the transient responses, as shown in Fig. 11(b). We hence expect to observe significant higher tracking errors for both the agents controlled by the direct-based scheme than those of the neuro-adaptive one (Figs. 11(c) and 11(d)). It is interesting to note that, despite the network subject to the coupled chaotic dynamics, the tracking errors of both the agents using the neuro-adaptive scheme converge almost to zero; we expected to observe the convergence of tracking errors to zero for the network subject to the coupled chaos.

The smoother control inputs, except within the small transient domains, in addition to the convergence of tracking errors to zero for the neuro-adaptive scheme extinguishing the coupled hyperchaos lead us to an easy selection between the methods. However, the computation time of the neuro-adaptive scheme is higher than that of the direct-based one for when the two agents are subject to the coupled hyperchaos: $t_{\text{direct}} = 0.981$ s and $t_{\text{neuro}} = 45.34$ s. We hence need to address the important trade-offs which include the smoother control inputs and smaller tracking errors of the neuro-adaptive and considerable lower computation time of the direct-based methods for suppressing the interconnected hyperchaotic dynamics.

5 Conclusions

Through this comparative research effort, we first represented the sixth-order interconnected analytical model of two bidirectional solenoid actuated butterfly valves operating in series. The initial conditions along with the critical parameters leading to the coupled chaotic and hyperchaotic dynamics were presented to be used in generating the energy-efficient desirable trajectories. The harmful responses were then characterized using the powerful tools of Lyapunov exponents and Poincaré map. We could reveal the coupled chaos and hyperchaos with the aid of one and two positive Lyapunov exponents, respectively, along with the irregular Poincaré maps of the two agents. We utilized both the decen-

tralized neuro- and direct-adaptive schemes for extinguishing both the coupled chaotic and hyperchaotic dynamics.

It was shown that, for the coupled chaos, the neuro-adaptive scheme yields practically feasible control inputs and subsequently the driving magnetic torques; the direct-based method leads to the chattering control inputs. Although the second trade-off, the computation time, can be helpful to properly select between the methods. Note that the computation time of the direct-based method is one-fiftieth of the neuro-adaptive one only for the two agents.

Surprisingly, the control inputs and subsequently the driving magnetic torques of both methods suppress the coupled hyperchaos are almost practically feasible; the neuro-adaptive method yields slightly smoother control inputs, except within the small transient domains, in comparison with the direct-based scheme. Also, the tracking errors of the neuro-adaptive scheme converge almost to zero, which may potentially outperform the direct-based method. However, the computational cost of the neuro-adaptive method is higher than that of the direct-based one.

References
